Answer the following questions. Calculators and mobile telephones are not allowed.

1. (5 points) Prove that

$$\sin(\tan^{-1} x - \cos^{-1} x) = \frac{x^2 - \sqrt{1 - x^2}}{\sqrt{x^2 + 1}}.$$

2. (5 points) Prove that

$$\coth 2x = \frac{\coth x + \tanh x}{2}$$

Evaluate the following integrals:

3. $\displaystyle\int_0^{\ln 2} x \sinh x\, dx$ (5 points)

4. $\displaystyle\int \frac{\sec^4 x}{1 - \sec^2 x}\, dx$ (5 points)

5. $\displaystyle\int \frac{\sqrt{x^2 - 4}}{x^2}\, dx$ (5 points)

6. $\displaystyle\int \frac{x^3 + x^2 + x - 1}{x^3 - x^2 + x - 1}\, dx$ (5 points)

7. $\displaystyle\int \frac{(x - 3)^2}{\sqrt{-x^2 + 6x - 5}}\, dx$ (5 points)

8. $\displaystyle\int \frac{1 + \sin x}{1 + \cos x}\, dx$ (5 points)